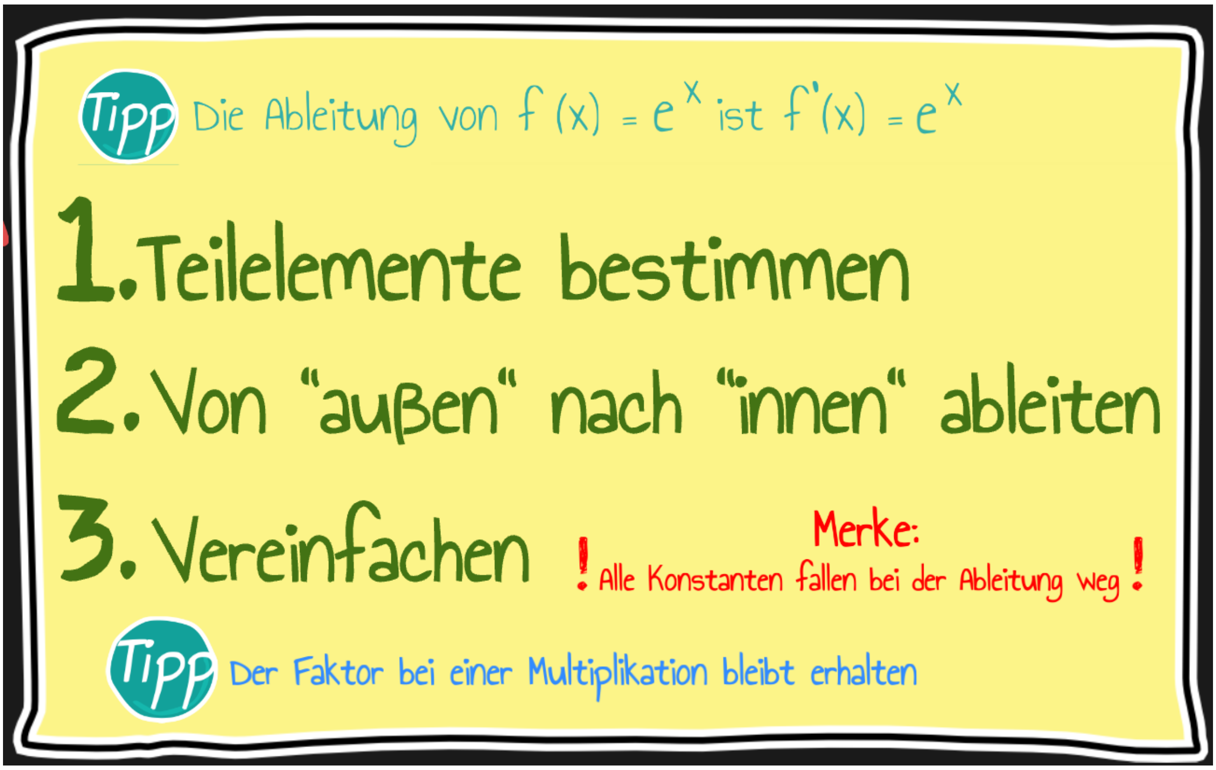
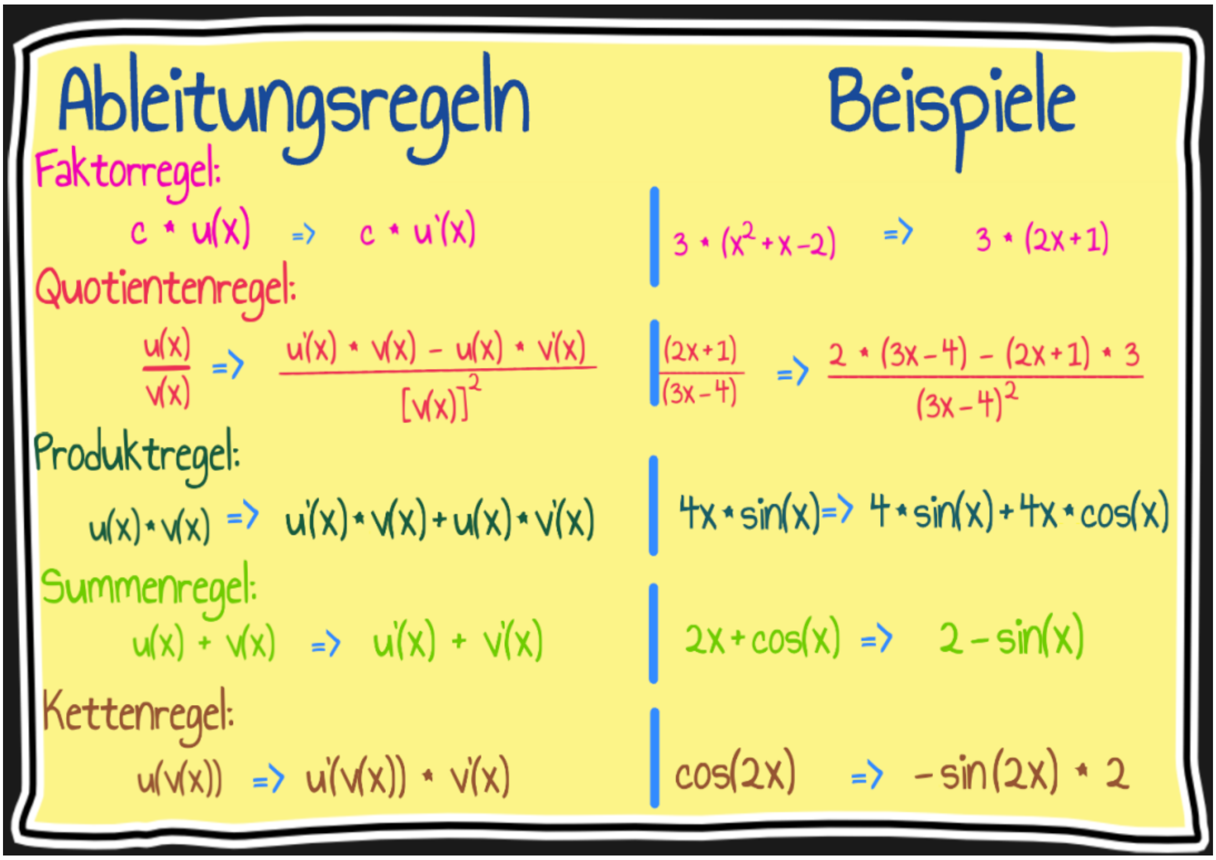
Lösungsblatt zur natürlichen Exponentialfunktion - Kettenregel

Das hast du schon gelernt:





Aufgabe 1

(1)

$f(x) = e^{4x+5}$

$f'(x) = 4 \cdot e^{4x+5}$

$f''(x) = 16 \cdot e^{4x+5}$

(2)

$f(x) = 2 \cdot e^{\frac{1}{2}x-3}$

$f'(x) = e^{\frac{1}{2}x-3}$

$f''(x) = \frac{1}{2} \cdot e^{\frac{1}{2}x-3}$

(3)

$f(x) = 0{,}5 \cdot e^{x^2}$

$f'(x) = 0{,}5 \cdot 2x \cdot e^{x^2}$

$= x \cdot e^{x^2}$

$u = x \qquad u' = 1$

$v = e^{x^2} \qquad v' = 2x \cdot e^{x^2}$

$f''(x) = 1 \cdot e^{x^2} + x \cdot 2xe^{x^2}$ $\qquad |e^{x^2}$ ausklammern

$= e^{x^2} \cdot (1 + 2x^2)$

(4)

$f(x) = 2{,}5 + e^{2x}$

$f'(x) = 2 \cdot e^{2x}$

$f''(x) = 4 \cdot e^{2x}$

(5)

$f(x) = 3x - 2 \cdot e^{3x}$

$f'(x) = 3 - 6e^{3x}$

$f''(x) = -18e^{3x}$

(6)

$f(x) = 2x^2 + 3x + 4e^{-2x}$

$f'(x) = 4x + 3 - 8e^{-2x}$

$f''(x) = 4 + 16e^{-2x}$

(7)

$f(x) = 4e^{2x} - 4e^{-2x}$

$f'(x) = 8e^{2x} + 8e^{-2x}$

$f''(x) = 16e^{2x} - 16e^{-2x}$

(8)

$f(x) = 2e^{3x-3} - 3e^{-2x+3}$

$f'(x) = 6e^{3x-3} + 6e^{-2x+3}$

$f''(x) = 18e^{3x-3} - 12e^{-2x+3}$

(9)

$f(x) = x^2 - 2e^{-x^2}$

$f'(x) = 2x + 4x \cdot e^{-x^2}$

$f''(x) = \ 2 + 4 \cdot e^{-x^2} + 4x \cdot e^{-x^2} \cdot (-2\mathrm{x})$

$\quad = 2 + (4 - 8x^2) \cdot e^{-x^2}$

(10)

$f(x) = x + e^{-2x + x^2} - e^{2x - x^2}$

$f'(x) = 1 + e^{-2x + x^2} \cdot (-2 + 2x) - e^{2x - x^2} \cdot (2 - 2x)$

$= 1 + (2x - 2)e^{-2x + x^2} + (2x - 2)e^{2x - x^2}\}$

Nebenrechnung:

$g(x) = (2x - 2) \cdot e^{-2x + x^2}$

$g'(x) = 2 \cdot e^{-2x + x^2} + (2x - 2) \cdot e^{-2x + x^2} \cdot (-2 + 2x)$ | Reihenfolge vertauschen

$= 2 \cdot e^{-2x + x^2} + (2x - 2)(2x - 2) \cdot e^{-2x + x^2} \cdot$

$= 2 \cdot e^{-2x + x^2} + (2x - 2)^2 \cdot e^{-2x + x^2}$

$= [2 + (2x - 2)^2] \cdot e^{-2x + x^2}$

$= [2 + 4x^2 - 8x + 4] \cdot e^{-2x + x^2}$

$= [4x^2 - 8x + 6] \cdot e^{-2x + x^2}$

Nebenrechnung:

$h(x) = (2x - 2) \cdot e^{2x - x^2}$

$h'(x) = 2 \cdot e^{2x - x^2} + (2x - 2) \cdot e^{2x - x^2} \cdot (2 - 2x)$ |(-1) ausklammern

$= 2 \cdot e^{2x - x^2} - (2x - 2)(2x - 2) \cdot e^{2x - x^2} \cdot$

$= 2 \cdot e^{2x - x^2} - (2x - 2)^2 \cdot e^{2x - x^2}$

$= [2 - (2x - 2)^2] \cdot e^{2x - x^2}$

$= [2 - (4x^2 - 8x + 4)] \cdot e^{2x - x^2}$

$= [2 - 4x^2 + 8x - 4] \cdot e^{2x - x^2}$

$= [-4x^2 + 8x - 2] \cdot e^{2x - x^2}$ | (-1) ausklammern

$= [4x^2 - 8x + 2] \cdot (-e^{2x - x^2})$

$$f''(x) = e^{-2x+x^2}(4x^2 - 8x + 6) - e^{2x-x^2}(4x^2 - 8x + 2)$$

(11)

$$f(x) = 2x^2 + x - \frac{1}{x} + e^{2x-2}$$
$$= 2x^2 + x - x^{-1} + e^{2x-2}$$

$$f'(x) = 4x + 1 - (-1) \cdot x^{-1-1} + 2e^{2x-2}$$
$$= 4x + 1 + x^{-2} + 2e^{2x-2}$$
$$= 4x + 1 + \frac{1}{x^2} + 2e^{2x-2}$$

$$f''(x) = 4 + (-2)x^{-2-1} + 4e^{2x-2}$$
$$= 4 - \frac{2}{x^3} + 4e^{2x-2}$$